



## Research Paper

## NORMALIZED DISTANCE LAPLACIAN ENERGY CHANGE DUE TO EDGE DELETION IN COMPLETE GRAPH AND COMPLETE MULTIPARTITE GRAPH

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## ABSTRACT

Let  $G$  be a connected graph with a distance matrix  $\mathcal{D}$ . The eigenvalues of  $\mathcal{D}$  forms the distance spectrum or  $\mathcal{D}$ -spectrum of  $G$ . The transmission of a vertex  $v$  in  $G$  is the sum of the distances from  $v$  to all other vertices of  $G$  and  $T(G)$  is the diagonal matrix with transmission of the vertices as diagonal entries. The distance Laplacian matrix is defined as  $\mathcal{D}^L(G) = T(G) - \mathcal{D}(G)$  and the normalized distance Laplacian matrix as  $\mathcal{D}^{\mathcal{L}}(G) = T(G)^{-1/2} \mathcal{D}^L(G) T(G)^{-1/2}$ . If  $\rho_1^{\mathcal{L}}, \rho_2^{\mathcal{L}}, \dots, \rho_n^{\mathcal{L}}$  are  $\mathcal{D}^{\mathcal{L}}$  eigenvalues of a graph  $G$ , then the normalized distance Laplacian energy is defined as  $\mathcal{D}^{\mathcal{L}}E(G) = \sum_{i=1}^n |\rho_i^{\mathcal{L}} - 1|$ . Let  $\mathcal{D}^{\mathcal{L}}E(G - e)$  be the normalized distance Laplacian energy when an edge  $e$  from a graph  $G$  is removed. In this paper we are analyzing the normalized distance Laplacian energy change due to an edge deletion.

## 1. INTRODUCTION

The spectral analysis of graphs, an essential area of research in algebraic graph theory, investigates the various spectra associated with graphs. A central problem in this field is determining which graphs can be uniquely identified by their spectra.

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